

ECE6930: Applied Fractional Calculus in Science and Engineering

Summer 2009 Course offering 8-Week Session Jun 08 - Jul 31, 2009

[Online registration information: SR 36347 ECE 6930 ST: APPLIED FRACTIONAL CALC, section 002.](#)
[\[3 credits; First lecture: June 15 Monday @ ENLAB 109, 12:00-17:00\]](#)

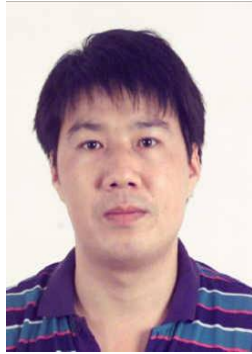
Why Fractional Calculus? Many real dynamic systems are better characterized using a non-integer order dynamic model based on fractional calculus or, differentiation or integration of non-integer order. Traditional calculus is based on integer order differentiation and integration. The concept of fractional calculus has tremendous potential to change the way we see, model, and control the nature around us. Denying fractional derivatives is like saying that zero, fractional, or irrational numbers do *not* exist.

How Fractional Calculus Will Benefit YOU? It will open new spectra of innovative research possibilities!! For example, in the control side, clearly, for closed-loop control systems, there are four situations. They are 1) IO (integer order) plant with IO controller; 2) IO plant with FO (fractional order) controller; 3) FO plant with IO controller and 4) FO plant with FO controller. From control engineering point of view, doing something better is the major concern. Existing evidences have confirmed that the best fractional order controller outperforms the best integer order controller. It has also been answered in the literature why to consider fractional order control even when integer (high) order control works comparatively well. Fractional order PID controller tuning has reached to a matured state of practical use. Since (integer-order) PID control dominates the industry, we believe FO-PID will gain increasing impact and wide acceptance. Furthermore, we also believe that based on some real world examples, fractional order control is ubiquitous when the dynamic system is of distributed parameter nature.

What you will be taught in this course:

- Introduction to Fractional Calculus and Special Functions Related to the Fractional Calculus.
- Fractional Derivatives and Integrals.
- Methods for the Solution of Fractional-order Differential Equations.
- Stability Analysis of Linear and Nonlinear Fractional-order Dynamic Systems.
- Numerical Simulation Algorithms for Fractional Control Systems and Fractional Order Signal Processing Using MATLAB/Simulink
- Research Opportunities and Challenges with Fractional Calculus

Introduction to four instructors



Dr Changpin Li is a professor of Mathematics Department of Shanghai University. His research interests include: applied theory and computation of bifurcation and chaos, the finite element method and calculation for fractional partial differential equations. He is an author of more than 40 referred papers and book chapters. He is also the Editor of Journal of Advanced Researches on Dynamical and Control Systems. He has been invited to visit City University of Hong Kong, Aden University, University of Pretoria, Nanyang Technological University. Now he is visiting CSOIS of USU as a Visiting Professor sponsored by CSC (China Scholarship Council).



Yan Li received his Ph.D. in applied mathematics in 2008 from Shandong University, China. From 2007 to 2008, he was studying as an exchange Ph.D. student in the ECE Dept. of Utah State University, which was supported by China Scholarship Council. His research interests include the theory of fractional calculus and its applications. Now he is visiting research fellow hosted by CSOIS of Utah State University.



Dingyu Xue received his doctorate in control engineering from Sussex University, UK in 1992. He joined the Northeastern University, China in 1993 and was employed as a professor in 1997. He is devoted to the work on MATLAB based education on mathematics, computer simulation and computer-aided control design, and published a few influential textbooks, two of which are in English. He is interested in the research of fractional-order control systems. He will be a visiting professor hosted by CSOIS from June to August 2009.



Dr YangQuan Chen is an Associate Professor of Electrical and Computer Engineering. His current areas of research interests include: distributed measurement and distributed control of distributed parameter systems using mobile actuator and sensor networks, mechatronics and controls (intelligent, optimal, robust, nonlinear and adaptive), applied fractional calculus, UAV cooperative control for remote sensing and real time water management and irrigation control. He holds 13 granted and 2 pending US patents. He is an author of two research monographs (Springer-Verlag 1999, 2007), 5 textbooks (SIAM Press 2007; Taylor & Francis/CRC 2008 and Tsinghua University Press 2002, 2004, 2007) and over 100 refereed journal papers.

Syllabus

Course Title:	Applied Fractional Calculus in Science and Engineering
Instructors:	Prof. Changpin Li (Lead-Instructor) Dr. Yan Li, (Co-Instructor) Prof. Dingyu Xue, (Co-Instructor) Prof. YangQuan Chen, (Co-Instructor)
Office:	EL256 (Prof. Changpin Li) EL204A (Dr. Yan Li) EL216 (Prof. YangQuan Chen) EL206 (Prof. Dingyu Xue)
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Office Hours:	M W F 10:00–11:30 Other hours by appointment.
Lecture Time:	Fridays 12:00–17:00 (June 15-July 31); but first lecture is on Monday; June 15, not June 10 The lecture time is divided into: 12:00-13:05, 13:15-14:20, 14:40-15:45 and 15:55-17:00.
Lecture Place:	ENLAB 109
Pre-requisites:	Classical Calculus and Ordinary Differential Equations.
Texts:	Podlubny I. <i>Fractional Differential Equations</i> . San Diego: Academic Press; 1999.
	Magin R. <i>Fractional Calculus in Bioengineering</i> . Begell House Inc., Redding; 2006.
	Kenneth S. Miller and Bertram Ross. <i>An Introduction to the Fractional Calculus and Fractional Differential Equations</i> . John Wiley and Sons, Inc., 1993.
	K.B. Oldham, J. Spanier: <i>Fractional Calculus: Theory and Applications, Differentiation and Integration to Arbitrary Order</i> . Academic Press, Inc., New York-London, 1974.
First Lecture	June 15, 2009. Monday. 12:00-5:00 (300 min. Total. Four 70 min sessions, three 7 min. breaks) Meet at EL109.

Final Exam:	12:30–14:00 on Tuesday, July 31. (subject to change)
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Course Summary:

The course is aimed at introducing the methods and tools of the fractional calculus into science and engineering. The emphasis is on big picture view, basic concepts, fundamental knowledge, hands-on problem solving abilities in fractional calculus.

Course Outcomes:

Students in the class will have ready basic knowledge on applying fractional calculus tools in research problems in science and engineering. In particular, students will learn basic computational techniques to solve basic fractional order control problems, fractional order signal processing problems.

Course Assessment:

Outcome	Assessment Method
Basic knowledge on applying fractional calculus tools in research problems in science and engineering	Mid term exam Final exam Homeworks
Basic computational techniques to solve basic fractional order control problems fractional order signal processing problems.	Computer assignments
Big picture view on applied fractional calculus	Literature review assignment

Course Outline:

- **Introduction to Fractional Calculus and Special Functions Related to Fractional Calculus.** (one week, Chen)
 - Introduction to the course, motivations from real world examples, big picture view of the field of fractional calculus and its research community.
 - Beginner's knowledge on special functions such as Gamma function, Mittag-Leffler function, Wright function etc.
- **Fractional Derivatives and Integrals.** (two weeks, Dr C. Li)
 - Grünwald-Letnikov Fractional Derivatives.
 - Riemann-Liouville Fractional Integrals,
 - Riemann-Liouville Fractional Derivatives.
 - Caputo Derivatives.
 - Generalized Fractional Derivatives.
 - Geometric and Physical Interpretation of Fractional Integration and Fractional Differentiation.
 - Sequential Fractional Derivatives.

- Left and Right Fractional Derivatives.
- Properties of Fractional Derivatives.
- Laplace Transforms of Fractional Derivatives.
- Fourier Transforms of Fractional Derivatives.
- Mellin Transforms of Fractional Derivatives.
- **Methods for the Solution of Fractional-order Differential Equations.** (two weeks, Dr C Li. Midterm quiz (the fifth week, one hour))
 - Fractional Green's Function.
 - The Mellin Transform Method.
 - Power Series Method.
 - Babenko's Symbolic Calculus Method.
 - Method of Orthogonal Polynomials.
 - Numerical Evaluation of Fractional Derivatives and Integrals.
 - The "Short-Memory" Principle.
 - Calculation of Heat Load Intensity Change in Blast Furnace Walls.
- **Stability Analysis of Fractional-order Differential Equations.** (one week, Dr Y Li)
 - Existence and Uniqueness Theorem as a Method of Solution.
 - Dependence of a Solution on Initial Conditions.
 - Fractional-order Linear System and its Stability Analysis in Time and Frequency Domains.
 - Linear Stability with Time-Delay.
 - Nonlinear Stability.
 - Some applications.
- Numerical Simulation Algorithms for Fractional Order Dynamic Systems and Fractional Order Signal Processing in MATLAB/Simulink (one week, Dr D Xue)
 - Discretization methods of fractional integrators/differentiators
 - Fractional order PID controller
 - Fractional order filters
- **Opportunities and Challenges in the research fields of fractional calculus** (3 hours, Dr Y Chen) (1 hour and a half, exam)
 - An overview of CSOIS Research Results on Applied Fractional Calculus
 - A case study on how to do original research when applying fractional calculus

SUPPLEMENTAL REFERENCES:

1. Podlubny, I., Heymans, N., Physical interpretation of initial conditions for fractional differential equations with Riemann-Liouville fractional derivatives. Rheologica Acta. vol. 45, 2006, pp. 765–771.
2. Podlubny I. Fractional-order systems and $PI^\lambda D^\mu$ –controllers, IEEE Transactions on Automatic Control, vol.44, no.1, January 1999, pp. 208-213.
3. Podlubny, I., Petras, I., Vinagre, B. M., O'Leary P., Dorcak L., Analogue realizations of fractional-order controllers. Nonlinear Dynamics, vol.29, no.1–4, 2002, pp.281–296.
4. Podlubny, I., Geometric and physical interpretation of fractional integration and fractional differentiation. Fractional Calculus and Applied Analysis, vol.5, no.4, 2002, pp.367–386.
5. Podlubny, I., Matrix approach to discrete fractional calculus. Fractional Calculus and

- Applied Analysis, vol.3, no.4, 2000, pp.359–386.
6. Magin, R.L., Fractional Calculus in Bioengineering, Critical Reviews in Biomedical Engineering, Part I 32(1):1-104, 2004, Part II 32(2):105-193, 2004, Part III 32(3-4):195-377.
 7. Mandelbrot, B.B., The fractal geometry of nature. New York: W. H. Freeman; 2000.
 8. Miller, K.S., Ross, B., An introduction to the fractional calculus. New York: John Wiley; 1993.
 9. Oldham, K.B., Spanier, J., The fractional calculus. New York: Academic Press; 1974.
 10. West, B.J., Bologna, M., Grigolini, P., Physics of fractal operators. New York: Springer; 2003.
 11. Deng, W.H., Li, C.P., Lu, J.H., Stability analysis of linear fractional differential system with multiple time-delays, Nonlinear Dynamics 48(4), 409-416, 2007.
 12. Deng, W.H., Li, C.P., Guo, Q., Analysis of fractional differential equations with multi-orders, Fractals 15(2), 173-182, 2007.
 13. Li, C.P., Deng, W.H., Chaos synchronization of fractional-order differential system, Int J Modern Phys B 20(7), 791-803, 2006.
 14. Sabatier, J., On Stability of Fractional Order Systems, in the proceedings of the 3rd IFAC Workshop on Fractional Differentiation and its Applications, Ankara, Turkey, 05 - 07 November, 2008.
 15. Li, Y., Chen, Y. Q., Podlubny, I., Cao, Y., Mittag-Leffler Stability of Fractional Order Nonlinear Dynamic Systems, in the proceedings of the 3rd IFAC Workshop on Fractional Differentiation and its Applications, Ankara, Turkey, 05 - 07 November, 2008.
 16. Diethelm, K., Ford, N.J., Analysis of Fractional Differential Equations, Journal of Mathematical Analysis and Applications 265(2), 229-248, 2002.
 17. Dingyu Xue* and YangQuan Chen (2008). *“Solving Applied Mathematical Problems with MATLAB”*. (ISBN : 978-1420082500) Chapman & Hall/CRC Press. Nov. 2008. (448 pages)
 18. Dingyu Xue, YangQuan Chen* and Derek Atherton (2007). *“Linear Feedback Control – Analysis and Design with Matlab”*. SIAM Press, 2007, ISBN: 978-0-898716-38-2. (348 pages) (Book review in IEEE Control Systems Magazine, Jan. 2009)

Website:

<http://fractionalcalculus.googlepages.com/>

<http://mechatronics.ece.usu.edu/csois/people/yan.li/>

<http://www.scicol.shu.edu.cn/Portals/149/lichangpin1215145159.htm>

Homework:

It is department pedagogical philosophy that students are responsible for their own learning. The instructor may not cover all of the material in each reading assignment in the lecture period. The student is therefore responsible for asking questions about reading material not covered in the lecture. Questions on exams may come from lectures, computer assignments, reading assignments, or supplementary materials given in class. Homework is due at the beginning of class on the due date. No late homework will be graded.

Grading: Scores will be weighted as follows:

Homework & computer assignments	30%
Literature Review	10%
Mid term exam	20%
Final exam	40%
Total	100%

Grades will be computed according to the following scale:

A	>93%
A-	>90%
B+	>87%
B	>84%
B-	>80%
C+	>77%
C	>74%
C-	>70%
D+	>67%
D	>64%
D-	>60%
F	<60%